APPENDIX 2

STEAM TURBINE DYNAMIC MODEL

THE DESIGN BASICS

The model is designed to give quantitatively correct responses to selected outside dynamic disturbances.

Steam turbine dynamical behaviour is captured through mathematical modelling of the three most relevant dynamic processes taking part in/around steam turbine. They are:

- Conservation of mass at turbine inlet;
- Conservation of mass (and heat) at turbine exhaust;
- Conservation of momentum at unit rotor (unit= turbine + generator or other driven machines as compressor or pump);

The above processes are modelled using:

- **Conservation equations**
  They balance accumulated mass, energy and momentum against sum of all the relevant entries and exits in/from the control volume. They normally result in various forms of differential equations. As an example there is (7) below;
- **Characteristic equations**
  Algebraic functions between relevant variables and parameters. Examples are (2) and (3).

The conservation processes are identified, simplified and mathematically modelled through the relevant conservation equations. The characteristics equations are then used to transform the conservation equations into forms that are suitable for numerical integration. The model response to outside disturbance is a result of numerical integration of the major conservation equation over time.

Some approximations have been accepted to simplify the identified mechanical processes. It has been done to reduce the amount of calculation. The main approximations are specified below:

- **Concentrated parameters (lumped parameter model);**
  The processes are simplified using concentrated parameters approach. It means that a single point represents all the points within control volume. This approach is commonly acceptable for modelling turbine dynamic behaviour.

- **Turbine thermodynamic model**
  Modelling turbine thermodynamic efficiencies and off- design “steam swallowing” ability is simplified from one in Ref. 2. Turbine stages are not calculated individually but all together as a block.

- **Steam thermodynamic properties**.
  - *enthalpy h[kJ/kg] and entropy s [kJ/kgK]*

h and s are calculated by a linear interpolation of the enthalpies and entropies at grid points. The grid points are calculated as per “Release on the IAPWS Formulation 1995”.

- Pressure grid points [bara]:
  0.00611, 0.01, 0.05, 0.1, 0.2, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0, 20.0, 30.0, 40.0, 50.0, 60.0, 70.0, 80.0, 90.0, 100.0, 110.0, 120.0, 200.00, 221.20
- Temperature grid points [^{0}C]:
  0.1, 5, 10, 30, 50, 70, 100, 200, 300, 400, 500, 600, 700, 800. For each pressure point there is also the corresponding saturated temperature taken as well. For example: 1 [bara] pressure has the following temperature grid points 99.63, 100, 200, 300, 400, 500, 600, 700, 800.
- Saturated steam enthalpy and entropy:
  Each pressure grid point has h and s as:
  For $x = 0$ h' and s'
  For $x = 1$ h'' and s''

- *Steam specific volume v [m^3/kg], i.e. specific density ρ [kg/m^3]*

$$v = \frac{1}{\rho} \quad (1)$$

Steam is considered as a “semi-perfect” gas. It satisfies the following equations:

$$pv = \frac{p}{\rho} = R_{approx}T \quad (2)$$

p [bara]	Steam pressure
T [K]	Steam temperature
R_{approx} [J/kgK]	Approximated steam gas constant calculated as per the equation: $R_{approx} = 450 - 1.4\,\{p\}_{bara}$ (3)

It is a known that steam does not behave as perfect gas in most of the range used in engineering practice. However, this approximation that significantly reduces the amount of calculation is considered as acceptable for this application.

- **Numerical integration method applied**
  Modified Euler method has been used for simulating the model in MSOffice VBA. It is rather fast but yields modest accuracy. For that reason the integration increment is set as short as 1/10 millisecond.

DYNAMIC AT TURBINE INLET – CONZERVATION OF MASS

The elementary single valve full arc steam admission model is presented here. Most of today’s units have partial arc steam admissions. They would normally have two to four partial admissions at the high pressure inlet. Partial arc is modelled by repeating the same calculation over for each valve and the adjacent part of the nozzles.

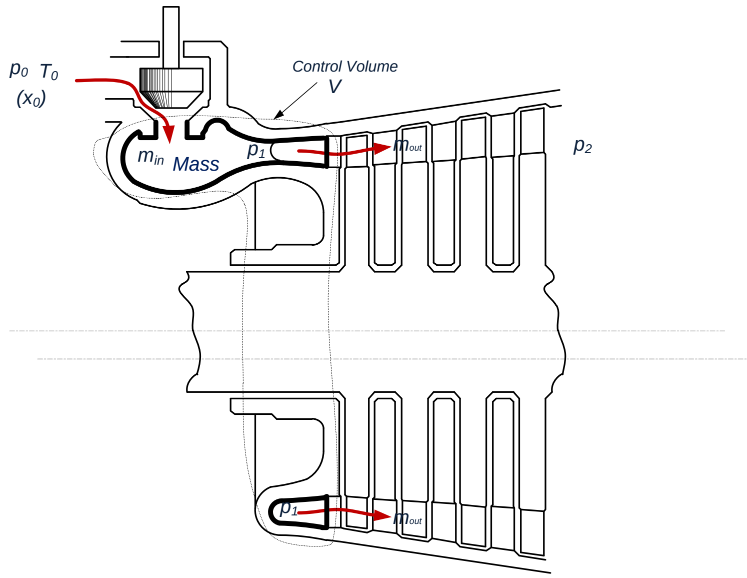

Figure 1

V [m^3] — The Control Volume being set around the space between the control valve and the entrance into the first stage nozzles. Often called turbine chest;

p_0 [bara] — Steam pressure at the control valve;

T_0 [°C] — Steam temperature at the control valve;

x_0 [-] — Steam phase. Alternative to t_0 in case of saturated steam;

p_1 [bara] — Steam pressure within turbine chest;

$Mass$ [kg] — Total steam mass contained in turbine chest;

m_{in} [kg/s] — Mass flow entering turbine chest. Modelled as steam flow through valve. Function of pressures p_0, p_1 and the control valve flow area A_{CV} [m_2]. Pressure ratio $\varepsilon = \left({}^{p_1}/_{p_0}\right)$ is compared against the critical pressure ratio $\varepsilon_{cr} = 0.56$. In case $\varepsilon \leq \varepsilon_{cr}$ the flow through the valve is calculated as the critical flow:

$$(m_{in})_{CR} = A_{cv}\, k \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} p_0\, \rho_0} \qquad (4)$$

In case $\varepsilon \geq \varepsilon_{cr}$ the flow is calculated as:

$$m_{in} = (m_{in})_{CR} \sqrt{1 - \frac{(\varepsilon - \varepsilon_{CR})^2}{(1-\varepsilon_{CR})^2}} \qquad (5)$$

k [-] — isentropic exponent

k = 1.3 → for superheated steam;

k = 1.135 →for saturated steam;

ρ_0 [kg/m^3] Steam density at the control valve;

p_2 [bara] Pressure at the turbine exhaust.

m_{out} [kg/s] Steam flow out from the chest. All other steam flows then one through the turbine are neglected. Modelled as flow through turbine at variable inlet and exhaust pressures. The expression applied here is a significantly simplified model however, the level of accuracy is considered as sufficient for the application.

$$m_{out} = D_0 \sqrt{\frac{p_1^2 - p_2^2}{p_{10}^2 - p_{20}^2}} \quad (6)$$

D_0 [kg/s] Rated turbine steam flow.

p_{10} [bara] Rated pressure at the first stage nozzles

p_{20} [bara] Rated pressure at turbine exhaust

Conservation of mass for the control volume V.

$$\frac{dMass}{dt} = \sum m_{in} - \sum m_{out} \quad (7)$$

$\sum m_{in}$ [kg/s] The sum of all mass flows entering turbine chest. With steam turbines there is normally only the flow through control valve per (5).

$$\sum m_{in} = m_{in} = (m_{in})_{CR} \sqrt{1 - \frac{(\varepsilon - \varepsilon_{CR})^2}{(1 - \varepsilon_{CR})^2}} \quad (8)$$

$\sum m_{out}$ [kg/s] The sum of all the mass flows from the chest. For this application all other flows except the one through the blades are neglected.

$$\sum m_{out} = m_{out} = D_0 \sqrt{\frac{p_1^2 - p_2^2}{p_{10}^2 - p_{20}^2}} \quad (9)$$

Equations (8), (9) express both m_{in} and m_{out} as functions of p_1. Equations (2) and (3) are used to express $Mass$ as function of p_1.

$$\frac{dMass}{dt} = \frac{dV\rho}{dt} = \frac{V}{TR_{approx}} \frac{dp}{dt} \quad (10)$$

The final form of the conservation equation:

$$\left(\frac{dp_1}{dt}\right)_t = \frac{[450 - 1.4(\{p_1\}_{bara})_{t-\Delta t}](T_1)_{t-\Delta t}}{V} \left[(m_{in})_{CR} \sqrt{1 - \frac{((\varepsilon)_{t-\Delta t} - \varepsilon_{CR})^2}{(1 - \varepsilon_{CR})^2}} - D_0 \sqrt{\frac{(p_1)_{t-\Delta t}^2 - (p_2^2)_{t-\Delta t}}{p_{10}^2 - p_{20}^2}})\right] \quad (11)$$

DYNAMIC AT TURBINE EXHAUST – CONSERVATION OF MASS AND HEAT

After passing through turbine the steam normally goes into heat exchanger, to be cooled and consequently condensated there. The exchanger would transfers the heat into either the environment or some other thermal process. Turbines exchanging heat with environment are called condensing turbines. They are primarily used to generate power. Turbines dumping their exhaust heat into another process are called backpressure turbines. From the design point of view the main difference between these two types is the exhaust pressure level and all that comes with it. With condensing turbines the exhaust pressure goes deep below the atmospheric and depends on the environment temperature. The main function of backpressure turbines is to provide heat while the generated power is a secondary product. Their exhaust pressure depends on the process that uses the exhaust heat.

On the process side both types are practically the same hence they are not distinguished as far the modelling is concerned.

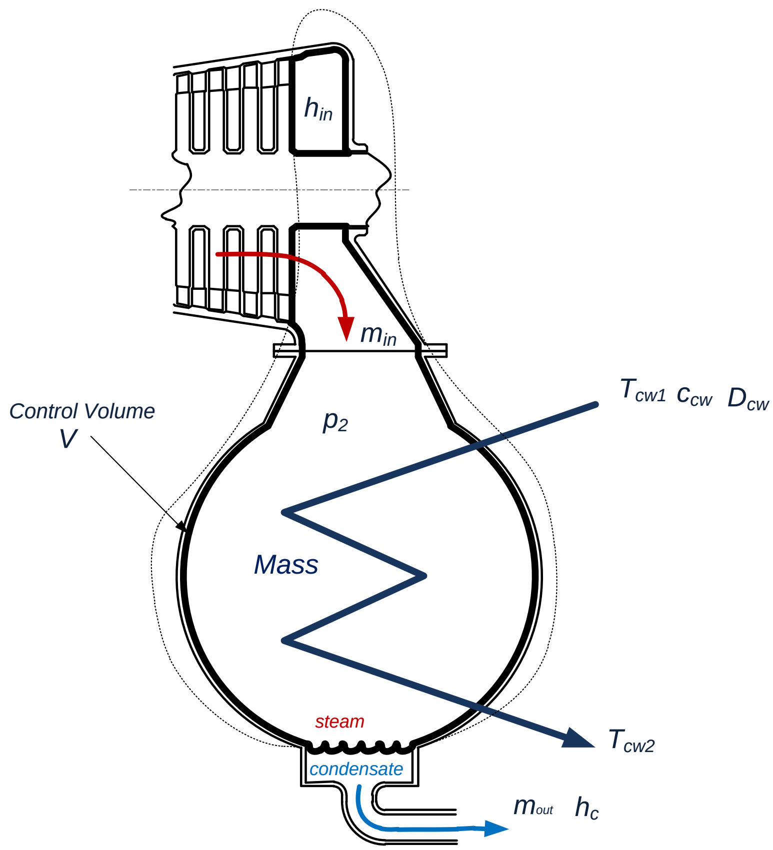

Figure 2

V [m^3] The Control Volume. Consists of turbine exhaust-hood together with the condenser/exchanger. The control volume is set to contain only steam with no condensate. For that purpose the bottom Control Volume boundary is imagined exactly at the surface between the steam and the condensate. This condition is easily acceptable as in the real world the condensate is evacuated automatically with a stable level being maintained by control means. Surface type exchanger is modelled i.e. cooling water circulates within tubes that are out of the control volume as far the mass balance is concerned;

p_2 [bara] Pressure at the turbine exhaust as well as in the whole condenser/exchanger;

$Mass$ [kg] Total steam mass contained in the control volume;

m_{in} [kg/s] Mass flow entering the control volume. It is the same as m_{out} calculated per (6);

h_{in} [kJ/kg] Enthalpy of the steam entering condenser/exchanger. Equal to h_2 calculated per (20) from chapter Turbine Thermodynamic here;

m_{out} [kg/s] Condensate being evacuated from the condenser/exchanger. As no accumulation of condensate has been considered m_{out} is equal to the amount of condensate being condensated;

h_c [kJ/kg] Enthalpy of the condensate being evacuated. This model assumes an ideal condenser/exchanger hence it does not consider condensate under-cooling that is normally present in real world. Hence in this model h_c is equal to h'_c i.e. the enthalpy of the saturated water under pressure p_2 .

D_{CW} [kg/s] Cooling water flow;

T_{CW1} [^{0}C] Cooling water inlet temperature;

T_{CW2} [^{0}C] Cooling water exit temperature;

c_{CW} [kJ/kg^{0}C] Cooling water heat capacity($c_{CW} = 4.19$);

Both conservation of mass and heat have been analyzed and combined. No heat accumulation has been allowed in the control volume. Hence the heat entering the control volume is the same as the exiting heat.

$$m_{out}(h_{in} - h_{out}) = D_{CW}\, c_{CW}\, (T_{CW2} - T_{CW1}) \qquad (12)$$

It results in the following equation that expresses m_{out}.

$$m_{out} = D_{CW} \quad c_{CW} \frac{((t_s)_{p_2} - t_{CW1})}{(h_2 - (h')_{p_2})} \qquad (13)$$

$(T_s)_{p_2}$ [^{0}C] Saturation temperature for pressure p_2 ;

h_2 [kJ/kg] enthalpy of turbine exhaust steam ;

$(h')_{p_2}$ [kJ/kg] enthalpy of the saturated water at pressure p_2 ;

Conservation of mass:

$$\frac{dMass}{dt} = m_{in} - m_{out} \tag{14}$$

$$\frac{dMass}{dt} = \frac{dV\rho}{dt} = \frac{V}{TR_{approx}} \frac{dp}{dt} = m_{in} - m_{out} \tag{15}$$

With m_{in} substituted by (9) and m_{out} substituted by (13) the final form of conservation equation comes as follows:

$$\left(\frac{dp_2}{dt}\right)_t =$$

$$\frac{[450-1.4(\{p_2\}_{bara})_{t-\Delta t}]\left((T_S)_{P_2}\right)_{t-\Delta t}}{V} \left[D_0 \sqrt{\frac{(p_1)^2_{t-\Delta t} - (p_2^2)_{t-\Delta t}}{p_{10}^2 - p_{20}^2}} - D_{CW} \;\; c_{CW} \frac{((t_S)_{p_2} - t_{CW1})}{(h_2 - (h')_{p_2})} \right]$$

(16)

DYNAMIC AT UNIT ROTOR – CONSERVATION OF MOMENTUM

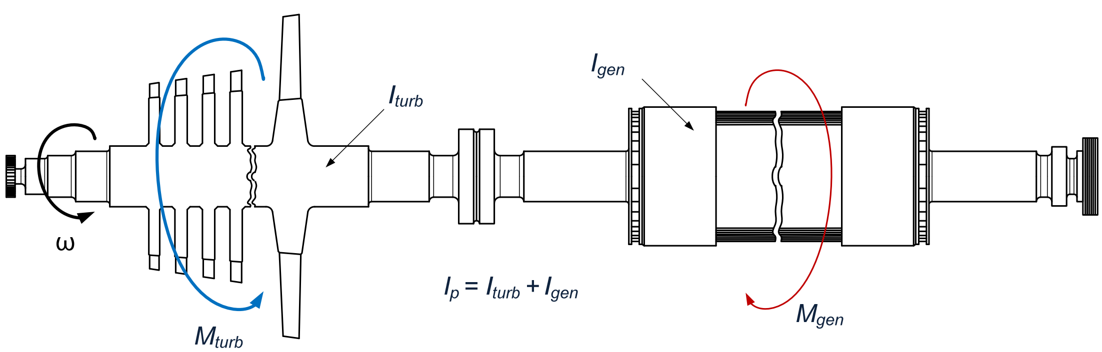

Figure 3

The basic conservation equation is:

$$\frac{dI_p\omega}{dt} = M_{turb} - M_{gen} \tag{17}$$

Using $\omega = \frac{2\pi RPM}{60}$ and $N = \omega M$:

$$\frac{dRPM}{dt} = \frac{900}{I_p \pi^2 RPM} \left(N_{turb} - N_{gen}\right) \tag{18}$$

$I_p [kgm^2]$	Rotor moment of inertia
RPM	Rotating speed in RPM
N_{turb} $[W]$	Turbine output
N_{gen} $[W]$	Generator load

Inertia is a mechanical property that is difficult to measure and quantify hence a term Unit Run-up Time $\left(\boldsymbol{t}_{Run-up}\right)$ is introduced to present it in a more convenient way. Unit Run-up Time is defined as a time required for unit to reach from zero to the rated speed with the turbine running full power with no generator load against it. This is an imaginative characteristic that cannot be figured out by measuring real world turbine.

The final form of the momentum conservation equation comes as follows.

$$\left(\frac{dRPM}{dt}\right)_t = \frac{RPM_0{}^2\,\pi}{60\,(N_{turb})_0\,\boldsymbol{t}_{Run-up}RPM}\left((N_{turb})_{t-\Delta t} - \left(N_{gen}\right)_{t-\Delta t}\right) \tag{19}$$

RPM_o	Rated unit RPM
$(N_{turb})_o\ [MW]$	Rated turbine output
$\boldsymbol{t}_{Run-up}$ [s]	Unit Run-up Time

TURBINE THERMODYNAMICS

Steam expansion through turbine and mechanical work obtained is calculated in steady state conditions.

Simplified four stages turbine is presented in Fig 4. A Mollier Chart presentation of the thermodynamic process taking part in the same turbine is in Fig. 5. The same characteristic points 0, 1 and 2 are noted in both Fig. 4 and Fig 5. The Mollier chart has an additional point. It is point 2_{iz}. It is an imaginative point that presents the end of the isentropic expansion through the turbine stages from point 1 to point 2. Point 2_{iz} does not exist in real world i.e. cannot be detected by direct measurements. Calculating point 2_{iz} is just a step toward determining point 2.

Turbine thermodynamic is modelled by calculating relevant steam properties at the characteristic points and processing them accordingly.

At the control valve steam is being throttled from pressures p_0 to pressure p_1. Throttling is normally an adiabatic process with no energy exchange ($h_0 = h_1$). Using that fact point 1 is being determined by calculating t_1 and s_1 as function of (p_1, h_0). In graphical sense that would be a horizontal line being drown from point 0 all the way to cross the pressure p_1 line.

The process between points 1 and 2 is an expansion through turbine stages. All stages are assumed to have the same isentropic efficiency (Eff_iz) so they can be modelled as a block. Point 2_{iz} is located by calculating h_{2iz} and x_{2iz} as functions of (p_2, s_1). In graphical sense that would mean a vertical line being drawn from point 1 all the way down to cross the pressure p_2 line. Enthalpy at point 2 is then calculated using the expression:

$$h_2 = h_1 - \frac{h_1 - h_{2iz}}{Eff\_iz} \tag{20}$$

After h_2 is calculated out then t_2 (x_2 is alternative to t_2 if point 2 is in the saturated region) can be calculated as function of (p_2, h_2).

At the end the turbine mechanical output is calculated as:

$$N_{turb} = D_0 \sqrt{\frac{p_1^2 - p_2^2}{p_{10}^2 - p_{20}^2}} \; (h_0 - h_2) \qquad (21)$$

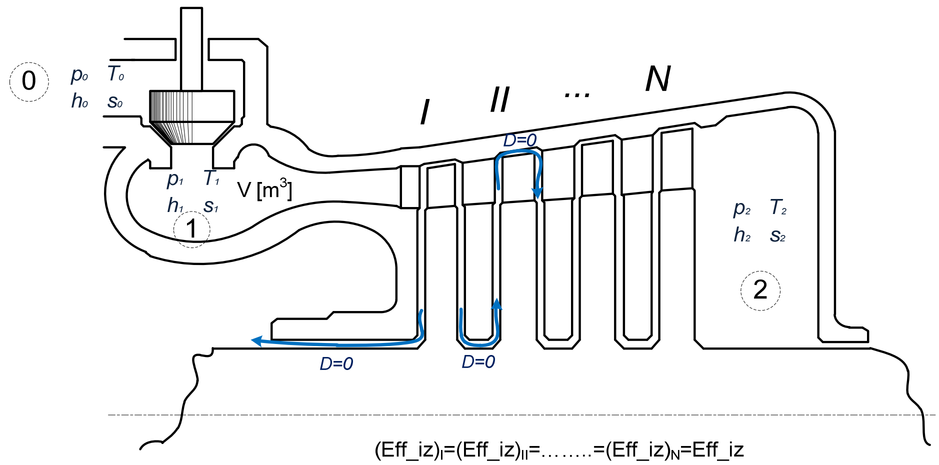

Figure 4

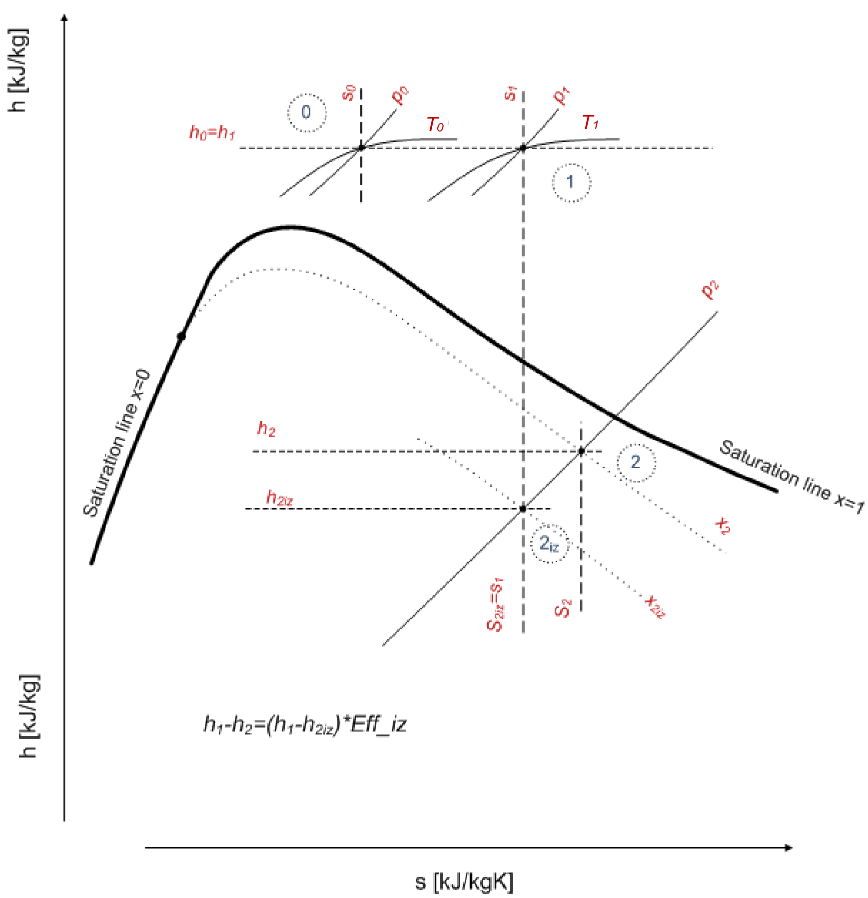

Figure 5

Ref. 2)

STEAM AND GAS TURBINES
Edited by A. Kostyuh and V. Frolov**.**
MIR Publishers - Moscow (1985)